FLUID MECHANICS
(BTME 301-18)

Unit 4: Fluid Dynamics

FLUID DYNAMICS

- A fluid in motion is subjected to several forces, which results in the variation of the acceleration and the energies involved in the flow of the fluid.
- The study of the forces and energies that are involved in the fluid flow is known as Dynamics of fluid flow.
- The various forces acting on a fluid mass may be classified as:
  - Body or volume forces
  - Surface forces
  - Line forces.

- **Body forces:** The body forces are the forces which are proportional to the volume of the body.

Examples: Weight, Centrifugal force, magnetic force, Electromotive force etc.

- **Surface forces:** The surface forces are the forces which are proportional to the surface area which may include pressure force, shear or tangential force, force of compressibility and force due to turbulence etc.
- **Line forces:** The line forces are the forces which are proportional to the length.

Example: surface tension.

- The dynamics of fluid flow is governed by Newton's second law of motion which states that the resultant force on any fluid element must be equal to the product of the mass and acceleration of the element and the acceleration vector has the direction of the resultant vector.
- The fluid is assumed to be in compressible and non- viscous.

$$\sum F_x = M.\ a$$

Where $\sum F$ represents the resultant external force acting on the fluid element of mass **M** and **a** is total acceleration.

- Both the acceleration and the resultant external force must be along same line of action.
- The force and acceleration vectors can be resolved along the three reference directions x, y and z and the corresponding equations may be expressed as ;

$$\sum F_x = M.\, a_x$$

$$\sum F_y = M.\, a_y$$

$$\sum F_z = M.\, a_z$$

FORCES ACTING ON FLUID IN MOTION

- The various forces that influence the motion of fluid are due to gravity, pressure, viscosity, turbulence and compressibility.
- The gravity force ‘Fg’ is due to the weight of the fluid and is equal to Mg . The gravity force per unit volume is equal to “ρ g”.
- The pressure force ‘Fp’ is exerted on the fluid mass, if there exists a pressure gradient between the two points in the direction of the flow.
- The viscous force ‘Fv’ is due to the viscosity of the flowing fluid and thus exists in case of all real fluids.
- The turbulent flow ‘Ft’ is due to the turbulence of the fluid flow.
- The compressibility force ‘Fc’ is due to the elastic property of the fluid and it is important only for compressible fluids.

FORCES ACTING ON FLUID IN MOTION

- If a certain mass of fluid in motion is influenced by all the above forces, then according to Newton's second law of motion
- The net force $F_x = M.\ a_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$
- If the net force due to compressibility(F_c) is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and the equation of motions are called **Reynolds's equations of motion.**

- For flow where (Ft) is negligible, the resulting equations of motion are known as **Navier – Stokes equation.**
- If the flow is assumed to be ideal, viscous force (F_v) is zero and the equations of motion are known as **Euler's equation of motion.**

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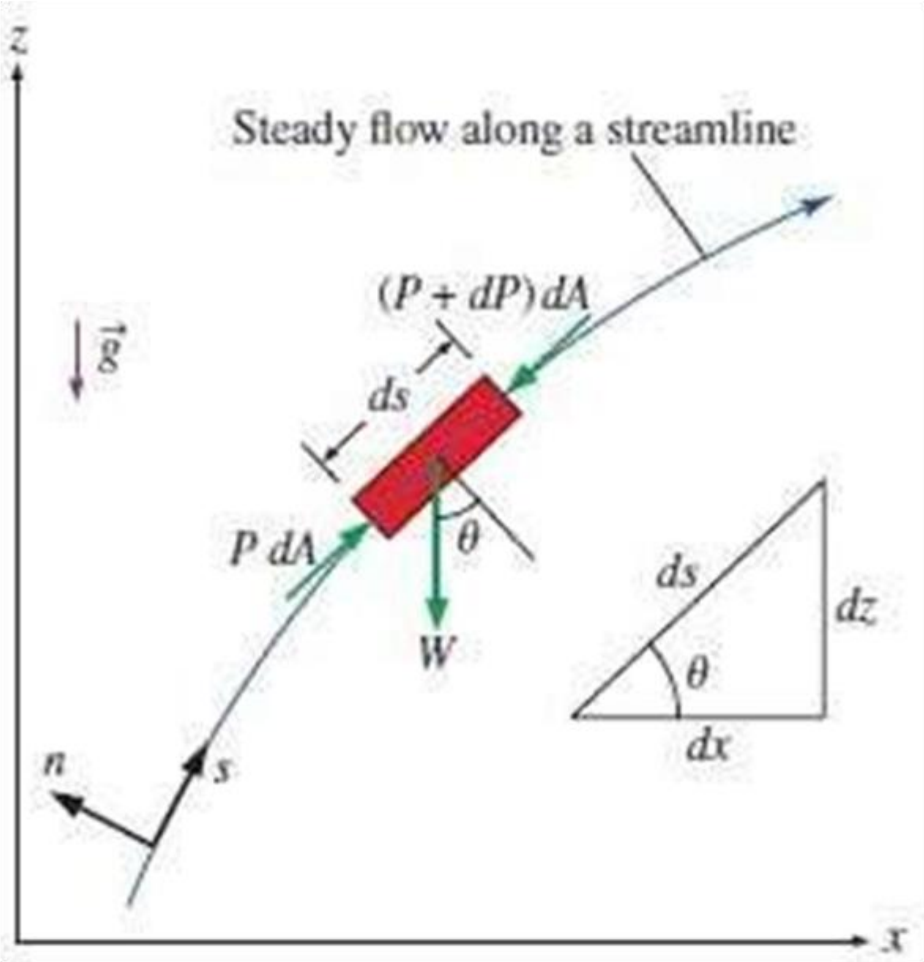
EULER'S EQUATION OF MOTION

- In this equation of motion the forces due to gravity and pressure are taken in to consideration.
- This is derived by considering the motion of the fluid element along a stream-line as:
- Consider a stream-line in which flow is taking place in s-direction.
- Consider a cylindrical element of cross-section dA and length



- The forces acting on the cylindrical element are:

➢ Pressure force pdA in the direction of flow.

➢ Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$

➢ Weight of element ρ g dA.ds

- Let θ is the angle between the direction of flow and the line of action of the weight of the element.
- The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element× acceleration in the direction of s.

$$pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta = \rho dA ds \times a$$

Whereas is the acceleration in the direction of s.

- Now, $a_s = \frac{dv}{dt}$ where 'v' is a function of s and t.

$$a_s = \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

- If the flow is steady, then $\frac{\partial v}{\partial t} = 0$. So, $a_s = \frac{v\partial v}{\partial s}$

$$-\frac{\partial p}{\partial s}dsdA - \rho g dAds\cos\theta = \rho dAds \times \frac{\partial v}{\partial s}$$

- Dividing by ρ dA.ds, $-\left(\frac{1}{\rho}\right) \times \left(\frac{\partial p}{\partial s}\right) - g\cos\theta = \frac{v\partial v}{\partial s}$

$$\left(\frac{1}{\rho}\right) \times \left(\frac{\partial p}{\partial s}\right) + g\cos\theta + \frac{v\partial v}{\partial s} = 0$$

$$\left(\frac{1}{\rho}\right) \times \left(\frac{\partial p}{\partial s}\right) + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = 0$$

BERNOULLI'S EQUATION

- Bernoulli's equation is obtained by integrating the Euler's equation of motion as

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = Constant$$

- If the flow is incompressible, ρ is constant and

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

MOMENTUM EQUATION

- It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass equal to the change in the momentum of the flow per unit time in that direction.
- The force acting on a fluid mass 'm' is given by Newton's second law of motion.

$$F = m \times a$$

- Where a is the acceleration acting in the same direction as force F.

MOMENTUM EQUATION

$$F = \frac{d\ mv}{dt}$$

The above equation is known as the momentum principle.

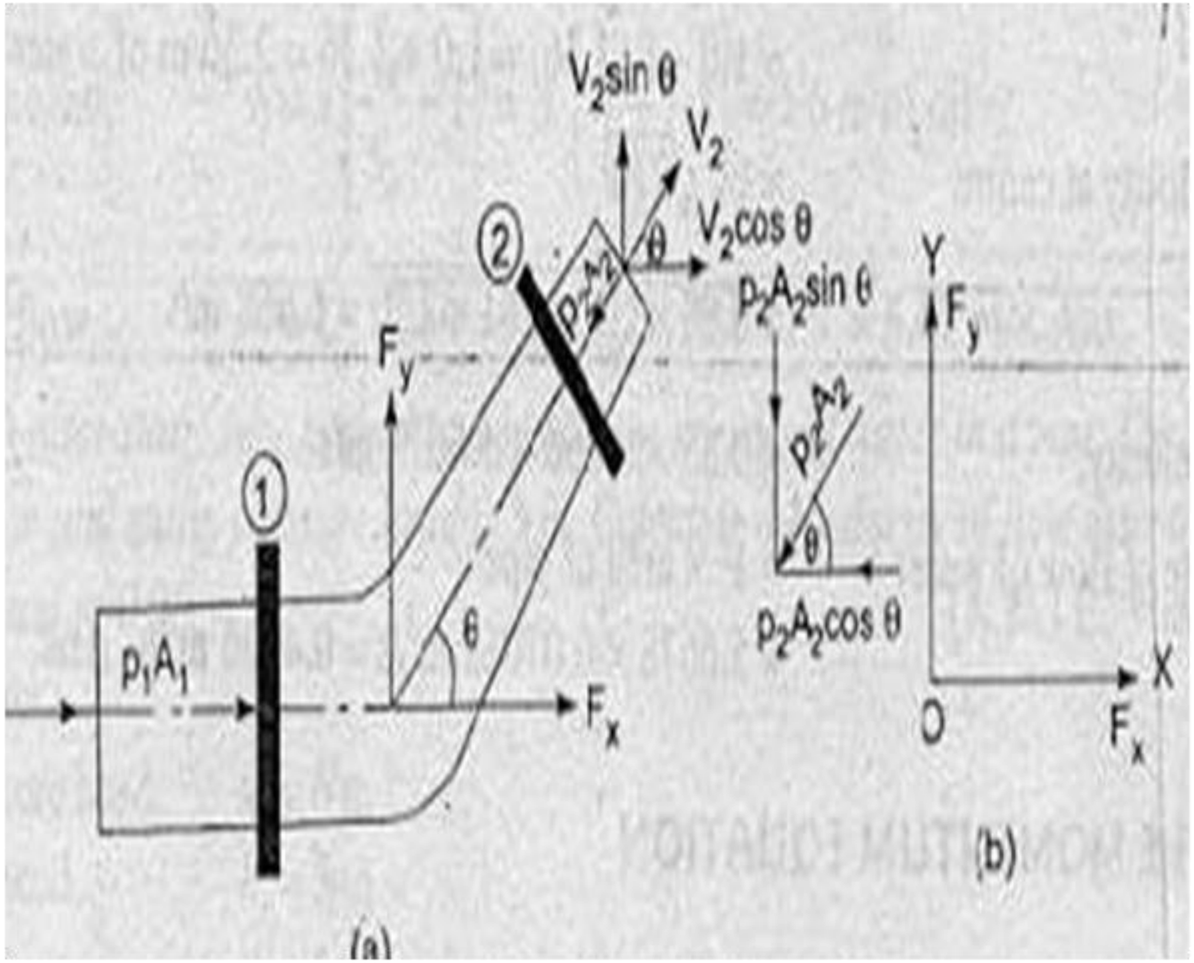
$$F.\,dt = d(mv)$$

The above equation is known as the impulse momentum equation.

- It states that the impulse of a force ‘F’ acting on a fluid mass m in a short interval of time ‘dt’ is equal to the change of momentum ‘d(mv)’ in the direction of force.

FORCE EXERTED BY A FLOWING FLUID ON A PIPE-BEND:

- The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.
- Consider two sections (1) and (2) as above
- Let v_1= Velocity of flow at section (1)
- P_1= Pressure intensity at section (1)
- A_1 = Area of cross-section of pipe at section (1)
- And V_2, P_2, A_2 are corresponding values of Velocity, Pressure, Area at section (2)



- Let F_x and F_y be the components of the forces exerted by the flowing fluid on the bend in *x* and *y* directions respectively.
- Then the force exerted by the bend on the fluid in the directions of *x* and *y* will be equal to F_X and F_Y but in the opposite directions.
- Hence the component of the force exerted by the bend on the fluid in the *x* - direction = - F_x and in the direction of *y* = - F_y.
- The other external forces acting on the fluid are $p_1 A_1$ and $p_2 A_2$ on the sections (1) and (2) respectively. Then the momentum equation in x-direction is given by
- Net force acting on the fluid in the direction of x = Rate of change of momentum in x -direction

$$= p_1 A_1 - p_2 A_2 \cos\theta - F_x = \text{(Mass per second) (Change of velocity)}$$

$= \rho Q$ (Final velocity in x-direction - Initial velocity in x-direction)

$= \rho\, Q\, (V_2 \cos\theta - V_1)$

$F_x = \rho\, Q\, (V_1 - V_2 \cos\theta) + p_1 A_1 - p_2 A_2 \cos\theta$ ________(1)

- Similarly the momentum equation in y-direction gives

$0 - p_2 A_2 \sin\theta - F_y = \rho\, Q\, (V_2 \sin\theta - 0)$

$F_y = \rho\, Q\, (-V_2 \sin\theta) - p_2 A_2 \sin\theta$ ______________(2)

- Now the resultant force (F_R) acting on the bend

$F_R = F_x^2 + F_y^2$

- And the angle made by the resultant force with the horizontal